MATHEMATICAL MODEL AND COMPUTATIONAL ANALYSIS OF SELECTED TRANSIENT STATES OF CYLINDRICAL LINEAR INDUCTION MOTOR FED VIA FREQUENCY CONVERTER

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Summary The mathematical model of cylindrical linear induction motor (C-LIM) fed via frequency converter is presented in the paper. The model was developed in order to analyze numerically the transient states. Problems concerning dynamics of ac-machines especially linear induction motor are presented in [1 – 7]. Development of C-LIM mathematical model is based on circuit method and analogy to rotary induction motor. The analogy between both: (a) stator and rotor windings of rotary induction motor and (b) winding of primary part of C-LIM (inductor) and closed current circuits in external secondary part of C-LIM (race) is taken into consideration. The equations of C-LIM mathematical model are presented as matrix together with equations expressing each vector separately. A computational analysis of selected transient states of C-LIM fed via frequency converter is presented in the paper. Two typical examples of C-LIM operation are considered for the analysis: (a) starting the motor at various static loads and various synchronous velocities and (b) reverse of the motor at the same operation conditions. Results of simulation are presented as transient responses including transient electromagnetic force, transient linear velocity and transient phase current.

1. INTRODUCTION

The mathematical model of C-LIM fed via frequency converter is referred to construction of C-LIM made in Department of Electrical Machines and Drives at Faculty of Electrical Engineering at Technical University of Częstochowa. The scheme of C-LIM structure being a base for development of mathematical model is depicted in Fig. 1. A real model of C-LIM is shown in Fig. 2 whereas inductor of C-LIM is shown in Fig. 3.

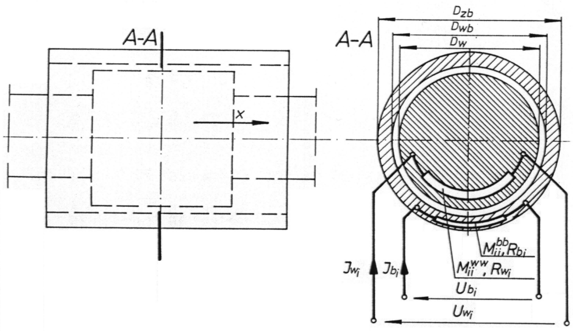

Fig. 1. Structure of cylindrical linear induction motor

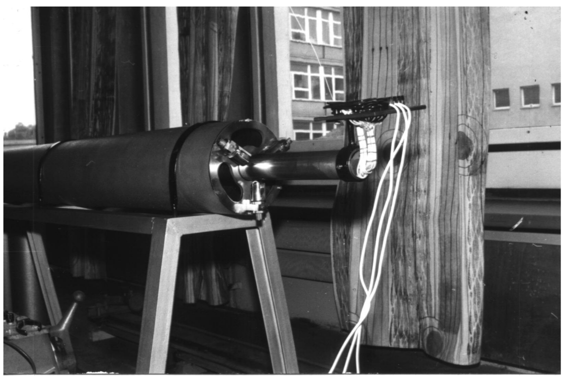

Fig. 2. General view of C-LIM

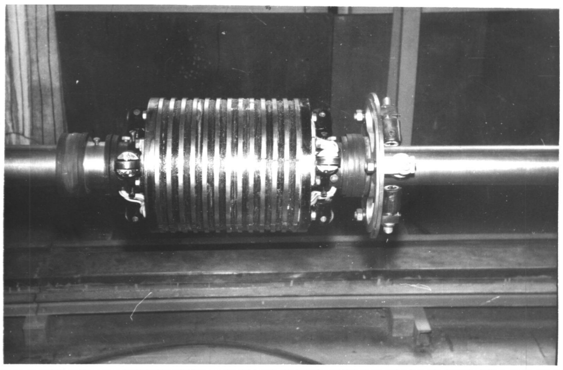

Fig. 3. Inductor of C-LIM

Mathematical model of C-LIM presented in this paper is a modified model presented in [3]. The aim of modification is possibility of simulating the transient states of C-LIM with inductor fed via frequency converter. In the paper [3] the mathematical model of C-LIM connected to the power grid was analyzed.

2. MATHEMATICAL MODEL

The following assumptions are taken into consideration in order to develop mathematical model of C-LIM: (a) the circuit correspondence between C-LIM inductor and cylindrical race of motor secondary part, (b) regularity of air-gap, (c) constant magnetic permeability of inductor and race magnetic circuits, (d) self-inductances and mutual inductances are independent of currents in C-LIM windings, (e) race resistance is independent of temperature, (f) the inductor moves along axis 0x, (g) a three-phase supply system fed C-LIM inductor is symmetrical.

It is assumed that “w” represents inductor (primary part of C-LIM) whereas “b” represents race (secondary part of C-LIM). The equations of electro-magnetic transient states in magnetically coupled circuits of C-LIM are expressed as the following matrix:

$$\begin{bmatrix} \boldsymbol{U}_w \\ \boldsymbol{U}_b \end{bmatrix} = \left\{ \begin{bmatrix} \boldsymbol{R}_w & \boldsymbol{N} \\ \boldsymbol{N} & \boldsymbol{R}_b \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \boldsymbol{M}_{ww} & \boldsymbol{M}_{wb} \\ \boldsymbol{M}_{bw} & \boldsymbol{M}_{bb} \end{bmatrix} \right\} \cdot \begin{bmatrix} \boldsymbol{I}_w \\ \boldsymbol{I}_b \end{bmatrix} \quad (1)$$

where respective matrix components are determined for i, j = 1, 2, 3 by the following dependences:

$$\boldsymbol{U}_w = [\underline{U}_{wi}]_{3\times1},\ \boldsymbol{I}_w = [\underline{I}_{wi}]_{3\times1},\ \boldsymbol{R}_w = [R_{wij}]_{3\times3}$$
$$\boldsymbol{U}_b = [\underline{U}_{bi}]_{3\times1},\ \boldsymbol{I}_b = [\underline{I}_{bi}]_{3\times1},\ \boldsymbol{R}_b = [R_{bij}]_{3\times3} \quad (2)$$
$$\boldsymbol{N} = [0]_{3\times3},\ \boldsymbol{M}_{ww} = [M^{ij}_{ww}]_{3\times3},\ \boldsymbol{M}_{bb} = [M^{ij}_{bb}]_{3\times3}$$
$$\boldsymbol{M}_{wb} = [M^{ij}_{wb}]_{3\times3},\ \boldsymbol{M}_{bw} = [M^{ij}_{bw}]_{3\times3}$$

where $R_{wij} = R_{wi}$ if $i = j$ and $R_{wij} = 0$ if $i \neq j$, $R_{bij} = R_{bi}$ if $i = j$ and $R_{bij} = 0$ if $i \neq j$.

A formula that expresses a C-LIM electromagnetic force F_e and equation of inductor motion are given as follows:

$$F_e = \frac{1}{2}\left[\boldsymbol{I}^T_w \quad \boldsymbol{I}^T_b\right] \cdot \frac{\partial}{\partial x} \begin{bmatrix} \boldsymbol{M}_{ww} & \boldsymbol{M}_{wb} \\ \boldsymbol{M}_{bw} & \boldsymbol{M}_{bb} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{I}_w \\ \boldsymbol{I}_b \end{bmatrix}$$

$$m\frac{dv}{dt} + Dv = F_e - F_o \quad (3)$$

where m, D, v, x are mass, dissipation constant, linear velocity and shift along the axis 0x of movable part of motor (inductor), $v = dx/dt$, F_o is load force.

The matrix $\boldsymbol{M}_{ww}$ of coefficients is given as follows:

$$\boldsymbol{M}_{ww} = L_{\sigma w}\boldsymbol{J} + 0{,}5 \cdot L_{\mu w} \cdot \boldsymbol{K}_1 \quad (4)$$

where $\boldsymbol{J} = [J_{ij}]_{3\times3}$, $J_{ij} = 1$ if $i = j$ and $J_{ij} = 0$ if $i \neq j$, $\boldsymbol{K}_1 = [K_{1ij}]_{3\times3}$, $K_{ij} = 2$ if $i = j$ and $K_{ij} = -1$ if $i \neq j$,

The eigenvalues of matrix $\boldsymbol{K}_1$ are determined using matrix $\boldsymbol{S}$ including eigenvectors in order to obtain diagonal matrix $\boldsymbol{S} \cdot \boldsymbol{K}_1 \cdot \boldsymbol{S}^{-1}$ of matrix $\boldsymbol{K}_1$ as well as diagonal matrix $\boldsymbol{D}_1$ of matrix $\boldsymbol{M}_{ww}$. The abovementioned transformations are explained below:

$$\boldsymbol{S} \cdot \boldsymbol{K}_1 \cdot \boldsymbol{S}^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix},\quad \boldsymbol{S} = \frac{1}{\sqrt{3}} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \quad (5)$$

where $a = e^{j2\pi/3}$. Diagonal matrix of matrix $\boldsymbol{M}_{ww}$ including inductor self-inductances and diagonal matrix of matrix $\boldsymbol{M}_{bb}$ including race self-inductances are expressed as $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$, respectively:

$$\boldsymbol{D}_1 = L_{\sigma w}\boldsymbol{J} + \frac{L_{\mu w}}{2} \cdot \boldsymbol{S} \cdot \boldsymbol{K}_1 \cdot \boldsymbol{S}^{-1} = L_{\sigma w}\boldsymbol{J} + \frac{3}{2} \cdot L_{\mu w} \cdot \boldsymbol{J}^{011}$$

$$\boldsymbol{D}_2 = L_{\sigma b}\boldsymbol{J} + \frac{3}{2} \cdot L_{\mu b} \cdot \boldsymbol{J}^{011} \quad (6)$$

where $L_{\mu w}, L_{\mu b}$ are magnetization inductances in inductor terms and race terms, $L_{\sigma w}, L_{\sigma b}$ are leakage inductances in inductor terms and race terms, $L_{\mu w} = L_{\mu b}\vartheta^2$, $L_{\sigma w} = L_{\sigma b}\vartheta^2$, ϑ is transformation ratio, $\boldsymbol{J}^{011} = [J^{011}_{ij}]_{3\times3}$ where $J^{011}_{ij} = 1$ if $i = j = 2,3$ and $J^{011}_{ij} = 0$ if $i = j = 1$ or $i \neq j$.

Matrix of mutual inductances between inductor and race are determined as follows:

$$\boldsymbol{M}_{wb} = |M_{wb}|' \cdot \begin{bmatrix} cosnx & cos(2\pi/3 + nx) & cos(4\pi/3 + nx) \\ cos(4\pi/3 + nx) & cosnx & cos(2\pi/3 + nx) \\ cos(2\pi/3 + nx) & cos(4\pi/3 + nx) & cosnx \end{bmatrix} \quad (7)$$

where $|M_{wb}|' = |M_{bw}|' = L_{\mu w}\vartheta^{-1}$, $n = \frac{\pi}{\tau}$, τ is pole pitch. For further transformations trigonometric functions in (7) are expressed by the following exponential functions according to dependences (8).

$$cos\, nx = \frac{1}{2}\left(e^{jnx} + e^{-jnx}\right)$$

$$cos\left(nx + \frac{2\pi}{3}\right) = \frac{1}{2}\left(ae^{jnx} + a^2e^{-jnx}\right) \quad (8)$$

$$cos\left(nx + \frac{4\pi}{3}\right) = \frac{1}{2}\left(a^2e^{jnx} + ae^{-jnx}\right)$$

Taking into consideration dependences (8), matrix of mutual inductances between inductor and race may be expressed as follows:

$$\boldsymbol{M}_{wb} = \frac{3}{2} \cdot |M_{wb}|' \cdot \boldsymbol{S}^{-1} \cdot \boldsymbol{E}_1 \cdot \boldsymbol{S},\quad \boldsymbol{E}_1 = [E^{ij}_1]_{3\times3} \quad (9)$$

where $E^{22}_1 = E^{33*}_1 = e^{jnx}$, $E^{ij}_1 = 0$ if $i = j = 1$ or $i \neq j$.

Diagonal matrixes $\boldsymbol{D}_3$ and $\boldsymbol{D}_4$ including mutual inductances between inductor and race as well as between race and inductor are expressed as follows:

$$\boldsymbol{D}_3 = \boldsymbol{S}\cdot\boldsymbol{M}_{wb}\cdot\boldsymbol{S}^{-1} = \frac{3}{2}\cdot\left|M_{wb}\right|'\cdot$$
$$\cdot\boldsymbol{S}\cdot\boldsymbol{S}^{-1}\cdot\boldsymbol{E}_1\cdot\boldsymbol{S}\cdot\boldsymbol{S}^{-1} = \frac{3}{2}\cdot\left|M_{wb}\right|'\cdot\boldsymbol{E}_1$$
$$\boldsymbol{D}_4 = \frac{3}{2}\cdot\left|M_{bw}\right|'\cdot\boldsymbol{E}_2, \quad \boldsymbol{E}_2 = \left[E_2^{ij}\right]_{3\times3} \tag{10}$$

where $E_2^{22} = E_2^{33*} = e^{-jnx}$, $E_1^{ij} = 0$ if $i = j = 1$ or $i \neq j$. Matrix equations (11) derived from system (1) are left-sided multiplied by transformation matrix $\boldsymbol{S}$. Moreover, the unit matrix $\boldsymbol{J} = \boldsymbol{S}^{-1}\cdot\boldsymbol{S}$ is added. The transformations are explained by system of equations (12). System of equations (12) after multiplication and development of matrixes takes a form (13).

$$\boldsymbol{U}_w = \boldsymbol{R}_w\cdot\boldsymbol{I}_w + \frac{d}{dt}\left(\boldsymbol{M}_{ww}\cdot\boldsymbol{I}_w\right) + \frac{d}{dt}\left(\boldsymbol{M}_{wb}\cdot\boldsymbol{I}_b\right)$$
$$\boldsymbol{U}_b = \boldsymbol{R}_b\cdot\boldsymbol{I}_b + \frac{d}{dt}\left(\boldsymbol{M}_{bw}\cdot\boldsymbol{I}_w\right) + \frac{d}{dt}\left(\boldsymbol{M}_{bb}\cdot\boldsymbol{I}_b\right) \tag{11}$$

$$\boldsymbol{S}\cdot\boldsymbol{U}_w = \boldsymbol{R}_w\cdot\boldsymbol{S}\cdot\boldsymbol{I}_w + \frac{d}{dt}\left(\boldsymbol{S}\cdot\boldsymbol{M}_{ww}\cdot\boldsymbol{S}^{-1}\cdot\boldsymbol{S}\cdot\boldsymbol{I}_w\right) +$$
$$+ \frac{d}{dt}\left(\boldsymbol{S}\cdot\boldsymbol{M}_{wb}\cdot\boldsymbol{S}^{-1}\cdot\boldsymbol{S}\cdot\boldsymbol{I}_b\right)$$
$$\boldsymbol{S}\cdot\boldsymbol{U}_b = \boldsymbol{R}_b\cdot\boldsymbol{S}\cdot\boldsymbol{I}_b + \frac{d}{dt}\left(\boldsymbol{S}\cdot\boldsymbol{M}_{bb}\cdot\boldsymbol{S}^{-1}\cdot\boldsymbol{S}\cdot\boldsymbol{I}_b\right) +$$
$$+ \frac{d}{dt}\left(\boldsymbol{S}\cdot\boldsymbol{M}_{bw}\cdot\boldsymbol{S}^{-1}\cdot\boldsymbol{S}\cdot\boldsymbol{I}_w\right) \tag{12}$$

$$\begin{bmatrix}\underline{U}_w^{(0)}\\ \underline{U}_w^{(1)}\\ \underline{U}_w^{(2)}\end{bmatrix} = \begin{bmatrix}R_{w1} & 0 & 0\\ 0 & R_{w2} & 0\\ 0 & 0 & R_{w3}\end{bmatrix}\cdot\begin{bmatrix}\underline{I}_w^{(0)}\\ \underline{I}_w^{(1)}\\ \underline{I}_w^{(2)}\end{bmatrix} + \frac{3L_{\mu w}}{2}\cdot\begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix}\cdot$$
$$\cdot\frac{d}{dt}\begin{bmatrix}\underline{I}_w^{(0)}\\ \underline{I}_w^{(1)}\\ \underline{I}_w^{(2)}\end{bmatrix} + \frac{3\left|M_{wb}\right|'}{2}\cdot\frac{d}{dt}\left\{\begin{bmatrix}0&0&0\\0&e^{jnx}&0\\0&0&e^{-jnx}\end{bmatrix}\cdot\begin{bmatrix}\underline{I}_b^{(0)}\\ \underline{I}_b^{(1)}\\ \underline{I}_b^{(2)}\end{bmatrix}\right\}$$
$$\begin{bmatrix}\underline{U}_b^{(0)}\\ \underline{U}_b^{(1)}\\ \underline{U}_b^{(2)}\end{bmatrix} = \begin{bmatrix}R_{b1} & 0 & 0\\ 0 & R_{b2} & 0\\ 0 & 0 & R_{b3}\end{bmatrix}\cdot\begin{bmatrix}\underline{I}_b^{(0)}\\ \underline{I}_b^{(1)}\\ \underline{I}_b^{(2)}\end{bmatrix} + \frac{3L_{\mu b}}{2}\cdot\begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix}\cdot \tag{13}$$
$$\cdot\frac{d}{dt}\begin{bmatrix}\underline{I}_b^{(0)}\\ \underline{I}_b^{(1)}\\ \underline{I}_b^{(2)}\end{bmatrix} + \frac{3\left|M_{bw}\right|'}{2}\cdot\frac{d}{dt}\left\{\begin{bmatrix}0&0&0\\0&e^{-jnx}&0\\0&0&e^{jnx}\end{bmatrix}\cdot\begin{bmatrix}\underline{I}_w^{(0)}\\ \underline{I}_w^{(1)}\\ \underline{I}_w^{(2)}\end{bmatrix}\right\}$$

Taking into consideration internal symmetry of primary and secondary parts of C-LIM as well as closed current circuits of race the following dependences (14) between selected variables of mathematical model for structure depicted in Fig. 1 are as follows:

$$R_{w1} = R_{w2} = R_{w3} = R_w;\; R_{b1} = R_{b2} = R_{b3} = R_b$$
$$\underline{U}_{b1} = \underline{U}_{b2} = \underline{U}_{b3} = 0;\; \underline{U}_b^{(0)} = \underline{U}_b^{(1)} = \underline{U}_b^{(2)} = 0 \tag{14}$$

Matrix equations (13) for each vector are as follows:

$$\underline{U}_w^{(0)} = R_w\underline{I}_w^{(0)} + L_{\sigma w}\frac{d}{dt}\underline{I}_w^{(0)},$$
$$\underline{U}_w^{(1)} = R_w\underline{I}_w^{(1)} + \left(L_{\sigma w} + \frac{3}{2}L_{\mu w}\right)\frac{d}{dt}\underline{I}_w^{(1)} +$$
$$+\frac{3}{2}\cdot\left|M_{wb}\right|'\cdot\frac{d}{dt}\left(\underline{I}_b^{(1)}\cdot e^{jnx}\right)$$
$$\underline{U}_w^{(2)} = R_w\underline{I}_w^{(2)} + \left(L_{\sigma w} + \frac{3}{2}L_{\mu w}\right)\frac{d}{dt}\underline{I}_w^{(2)} +$$
$$+\frac{3}{2}\cdot\left|M_{wb}\right|'\cdot\frac{d}{dt}\left(\underline{I}_b^{(2)}\cdot e^{-jnx}\right)$$
$$0 = R_b\underline{I}_b^{(0)} + L_{\sigma b}\frac{d}{dt}\underline{I}_b^{(0)}, \tag{15}$$
$$0 = R_b\underline{I}_b^{(1)} + \left(L_{\sigma b} + \frac{3}{2}L_{\mu b}\right)\frac{d}{dt}\underline{I}_b^{(1)} +$$
$$+\frac{3}{2}\cdot\left|M_{bw}\right|'\cdot\frac{d}{dt}\left(\underline{I}_w^{(1)}\cdot e^{-jnx}\right)$$
$$0 = R_b\underline{I}_b^{(2)} + \left(L_{\sigma b} + \frac{3}{2}L_{\mu b}\right)\frac{d}{dt}\underline{I}_b^{(2)} +$$
$$+\frac{3}{2}\cdot\left|M_{bw}\right|'\cdot\frac{d}{dt}\left(\underline{I}_w^{(2)}\cdot e^{jnx}\right)$$

Two equations of the system (15) including dependences between zero-sequence variables are independent of other equations and will be omitted in further analysis.

In general case the components of three-phase supply voltage may be expressed as follows:

$$\underline{U}_1 = \frac{1}{2}\cdot\sum_{i=1}^{\infty}\left\{U_{mi}\left(e^{j\gamma_i} + e^{-j\gamma_i}\right)\right\} = \sum_{i=1}^{\infty}\left\{U_{mi}\cos\gamma_i\right\}$$
$$\underline{U}_2 = \frac{1}{2}\cdot\sum_{i=1}^{\infty}\left\{U_{mi}\left(e^{j(\gamma_i-2\pi/3)} + e^{-j(\gamma_i-2\pi/3)}\right)\right\} =$$
$$= \sum_{i=1}^{\infty}\left\{U_{mi}\cos\left(\gamma_i - 2\pi/3\right)\right\} \tag{16}$$

$$\underline{U}_3 = \frac{1}{2} \cdot \sum_{i=1}^{\infty} \left\{ U_{mi} \left(e^{j(\gamma_i - 4\pi/3)} + e^{-j(\gamma_i - 4\pi/3)} \right) \right\} =$$
$$= \sum_{i=1}^{\infty} \left\{ U_{mi} \cos(\gamma_i - 4\pi/3) \right\}$$

$$\gamma_i = \int_0^t \omega_i d\lambda + \varphi_i \ , \quad \omega_i = i\omega_1$$

where U_{mi}, ω_i, φ_i are amplitude, angular frequency and initial phase for i-th harmonic of supply voltage, ω_1 is angular frequency for fundamental harmonic. The zero-sequence, positive-sequence and negative-sequence components for voltages defined by dependences (16) are given as the following matrix equation:

$$\begin{bmatrix} \underline{U}^{(0)} \\ \underline{U}^{(1)} \\ \underline{U}^{(2)} \end{bmatrix} = \boldsymbol{S} \cdot \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_3 \end{bmatrix} = \frac{\sqrt{3}}{2} \cdot \sum_{i=1}^{\infty} \left\{ U_{mi} \cdot \begin{bmatrix} 0 \\ e^{j\gamma_i} \\ e^{-j\gamma_i} \end{bmatrix} \right\} \quad (17)$$

It results from (17) that $\underline{U}^{(1)}$ and $\underline{U}^{(2)}$ are coupled each other, i.e. $\underline{U}^{(1)} = \underline{U}^{(2)*}$. Thus, only positive-sequence component is chosen for further transformations:

$$\underline{U}_w^{(1)} = \underline{U}^{(1)} = \frac{\sqrt{3}}{2} \cdot \sum_{i=1}^{\infty} \left\{ U_{mi} \cdot e^{j\gamma_i} \right\}$$

The system of equations (18) is obtained as a result of transformation of race variables to the inductor coordinates.

$$\begin{aligned} \underline{U}_w^{(1)} &= R_w \underline{I}_w^{(1)} + L_w \cdot \frac{d}{dt} \underline{I}_w^{(1)} + M_{wb} \cdot \frac{d}{dt} \left(\underline{I}_b^{'(1)} \cdot e^{jnx} \right) \\ 0 &= R_b^{'} \underline{I}_b^{'(1)} + L_b^{'} \cdot \frac{d}{dt} \underline{I}_b^{'(1)} + M_{wb} \cdot \frac{d}{dt} \left(\underline{I}_w^{(1)} \cdot e^{-jnx} \right) \end{aligned} \quad (18)$$

where $L_w = L_{\sigma w} + \frac{3}{2} L_{\mu w}$, $L_b^{'} = \left(L_{\sigma b} + \frac{3}{2} L_{\mu b} \right) \vartheta^2$, $M_{wb} = \frac{3}{2} L_{\mu w}$, $R_b^{'} = R_b \vartheta^2$, $\underline{I}_b^{'(1)} = \underline{I}_b^{(1)} \cdot \vartheta^{-1}$.

Equivalent variable $\underline{I}_b^{'} = \underline{I}_b^{'(1)} \cdot e^{jnx}$ is applied in order to transform the currents conducted in closed circuits of race to the reference system connected to the inductor. Moreover, the equivalent variables $\underline{I}_w = \underline{I}_w^{(1)}$ and $\underline{U}_w = \underline{U}_w^{(1)}$ are applied in order to simplify notation. The equations of electromagnetic transient states expressed in reference system connected to the inductor are obtained. The equations together with motion equation form mathematical model of C-LIM described by the following system of equations:

$$\begin{aligned} \underline{U}_w &= R_w \underline{I}_w + L_w \cdot \frac{d}{dt} \underline{I}_w + M_{wb} \cdot \frac{d}{dt} \underline{I}_b^{'} \\ 0 &= R_b^{'} \underline{I}_b^{'} + L_b^{'} \cdot \frac{d}{dt} \underline{I}_b^{'} + M_{wb} \cdot \frac{d}{dt} \underline{I}_w - \\ &\quad - jnv \left(L_b^{'} \underline{I}_b^{'} + M_{wb} \underline{I}_w \right) \\ m \frac{dv}{dt} &= jnM_{wb} \left(\underline{I}_w^{*} \underline{I}_b^{'} - \underline{I}_w \underline{I}_b^{'*} \right) - F_{obc} \end{aligned} \quad (19)$$

The equations of mathematical model allow analyzing the transient states of C-LIM. As a result of the analysis the transient responses and trajectories for various real loads and various supply systems may be obtained.

3. COMPUTATIONAL ANALYSIS

Computational analysis of selected transient states of C-LIM was made using a mathematical model of C-LIM fed via frequency converter. Technical data and parameters of C-LIM constructed at Department of Electrical Machines and Drives at Faculty of Electrical Engineering at Technical University of Częstochowa are taken into consideration.

Computational analysis is based on the system of equations (19). The system (19) includes equation of inductor electric circuit, equation of race electric circuit written in inductor terms and equation of motion. The formulas (16) that determine in general form components of three-phase supply voltage are taken into consideration. The system of equations describing operation of C-LIM in transient states was solved using own computational program written in language C.

The following working conditions of C-LIM defined by load force F_o and frequency f of inverter voltage were considered in order to analyze computationally the C-LIM transient states:

1. Starting the C-LIM at F_o = 0N, 100N, 200N and f = 30Hz, 40Hz, 50Hz, respectively.
2. Reverse of the C-LIM at F_o = 0N, 100N, 200N and f = 30Hz, 40Hz, 50Hz, respectively.

A self-load necessary to unbalance the inductor of C-LIM is assumed to be [3]. The self-load is equal to friction force of F_t = 23N. Examples of transient responses of started C-LIM using frequency converter are shown in Fig. 4 and 5, respectively. Examples of transient responses of reversed C-LIM using frequency converter are shown in Fig. 6 and 7.

Selected parameters of transient force $F_e = F(t)$, transient velocity $v = v(t)$ and transient current $i_1 = i(t)$ of started motor at various frequencies of inverter voltage and various loads of the motor are presented in Table 1, where f is frequency of inverter voltage, F_o is load force, F_{emax} is maximal amplitude of electromagnetic force, ΔF_e is amplitude of steady-state oscillation of electromagnetic force, t_s is time of stabilization of velocity, v_s is steady-state velocity.

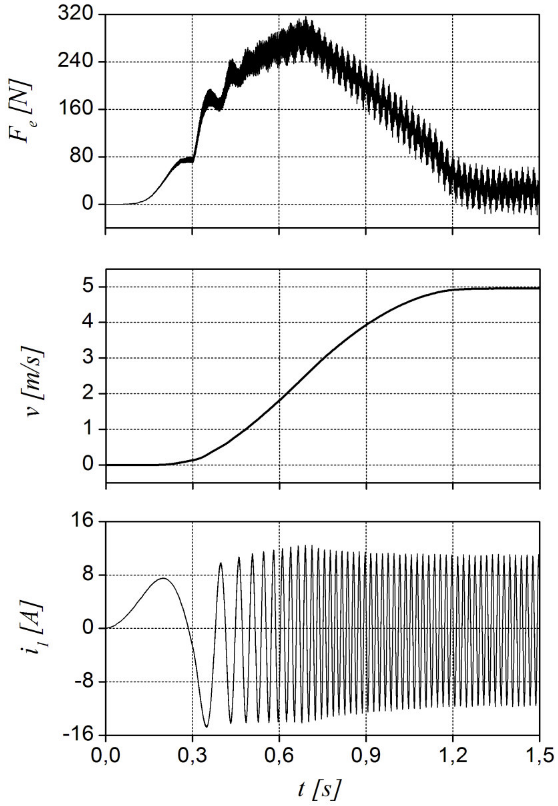

Fig. 4. Transient responses of started C-LIM using frequency converter at F_o = 0N and f = 50Hz, where $F_e = F(t)$ is electromagnetic force, $v = v(t)$ is linear velocity and $i_1 = i(t)$ is phase current

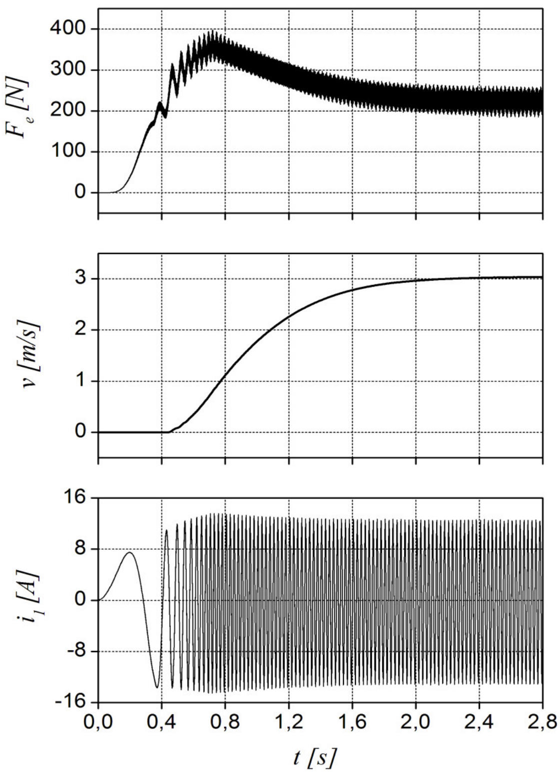

Fig. 5. Transient responses of started C-LIM using frequency converter at F_o = 200N and f = 40Hz

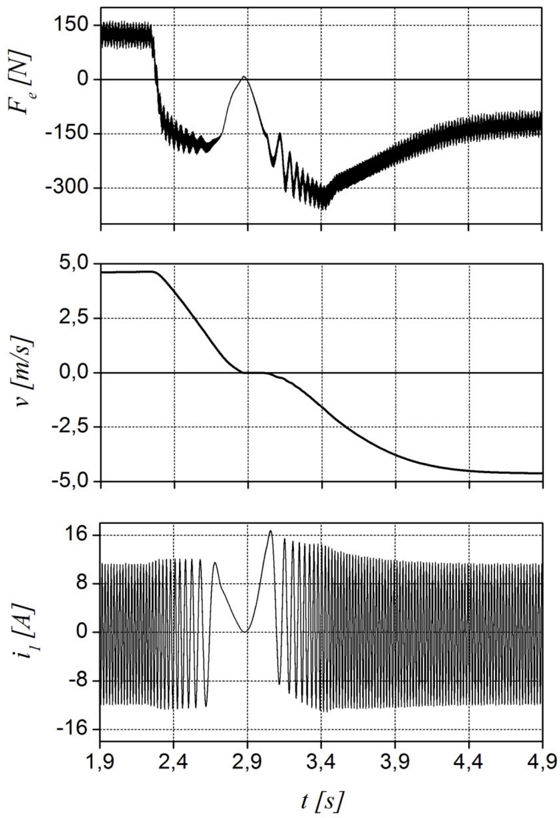

Fig. 6. Transient responses of reversed C-LIM using frequency converter at F_o = 100N and f = 50Hz

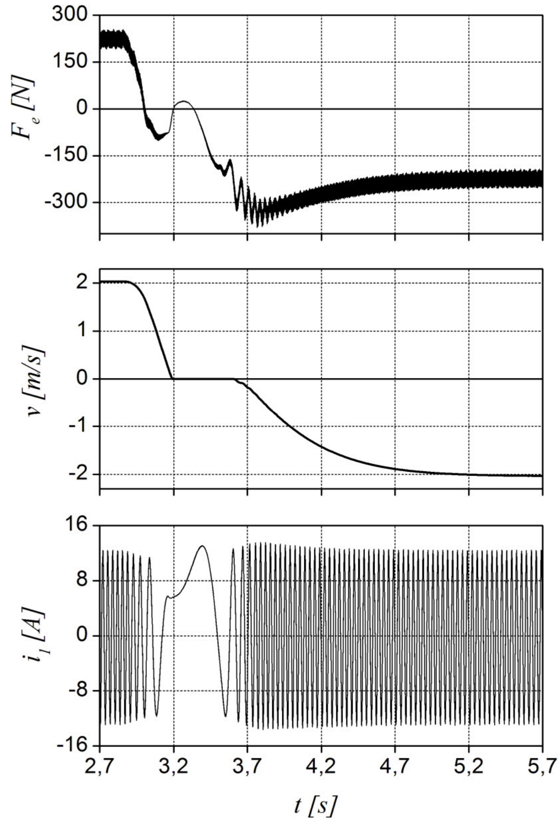

Fig. 7. Transient responses of reversed C-LIM using frequency converter at F_o = 200N and f = 30Hz

Selected parameters of transient force $F_e = F(t)$, transient velocity $v = v(t)$ and transient current $i_1 = i(t)$ of reversed motor are presented in Table 2, where v_r is steady-state velocity, t_r is time of transition between steady-states velocities, t_0 is time of immobilization of inductor.

Tab. 1. Parameters of transient responses of started motor

Lp.	f [Hz]	F_o [N]	F_{emax} [N]	ΔF_e [N]	t_s [s]	v_s [m/s]
1	2	3	4	5	6	7
1.	50	0	305	45	1,25	4,96
		100	375	170	2,20	4,62
		200	380	210	3,20	3,32
2.	40	0	300	40	0,98	3,96
		100	370	165	1,75	3,74
		200	390	215	2,50	3,04
3.	30	0	265	40	0,72	2,95
		100	330	165	1,42	2,73
		200	375	210	2,20	2,03

Tab. 2. Parameters of transient responses of reversed motor

Lp.	f [Hz]	F_o [N]	F_{emax} [N]	v_r [m/s]	t_r [s]	t_0 [s]
1	2	3	4	5	6	7
1.	50	0	320	-4,96	2,00	0
		100	365	-4,62	2,85	0,30
		200	385	-3,38	4,62	0,53
2.	40	0	310	-3,96	1,30	0
		100	335	-3,74	2,20	0,29
		200	405	-3,04	3,25	0,52
3.	30	0	275	-2,96	1,10	0
		100	330	-2,73	1,96	0,26
		200	385	-2,03	2,75	0,48

In Table 2, maximal amplitude of force F_e is an absolute value. Time of transition between steady-states velocities is measured from positive to negative steady-state velocity whereas time of immobilization of inductor is measured as time of overlapping the transient velocity $v = v(t)$ with level of $v = 0$.

4. CONCLUSION

Taking into consideration simulated transient states of started and reversed C-LIM as well as selected parameters presented in Table 1 and Table 2, it may be concluded that:
- the maximal difference in oscillation of electromagnetic force F_e in comparison to steady-state oscillation appears during starting the non-loaded C-LIM ($F_o = 0$) independently of frequency of inverter voltage,
- transient responses of C-LIM in the range of starting operation are similar to transient responses of rotary induction motor,
- time of transition between steady-states velocities at given frequency of inverter voltage increases together with increase of load; a ratio of mass density of inductor together with suspension system to load force is a prime of importance; this topic was not of concern in this paper,
- the oscillation of simulated transient phase current in inductor windings is minor during starting and reverse of the motor; thus, this oscillation was not studied in the paper,
- time of immobilization of inductor during reverse of the motor increases together with increase of load; however, length of mentioned time is minor (below 1 second); it causes that this quantity is negligible in the range of usage of driving systems with application of C-LIM.

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